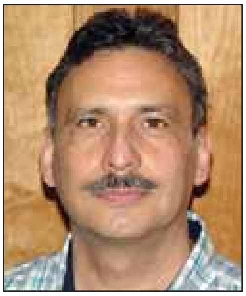
Robert X. Perez



Affinity Laws for Pumping Systems (ALPS) – Part Two

Conclusion of our look at a new tool that predicts how pumps and systems interact.

Last month, I opened this discussion by explaining how the affinity laws are used to estimate the effect of speed and impeller diameter changes on pump performance.

I emphasized that these laws convey simple relationships that are only meant to be used on pump curve predictions, or when system curves fit the general form of $H = KQ^2$, i.e. friction only system curves! Using them to directly determine the effects impeller or speed changes for system curves with static head can result in serious calculation errors.

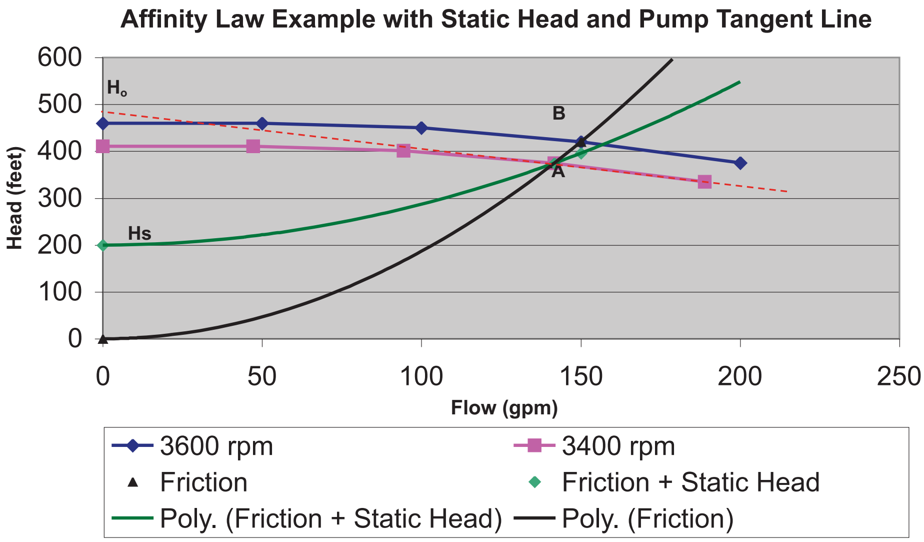


Figure 3

Description	Symbol	Value	Units
Vertical axis intersection of pump curve tangent line at point Q_1, H_1	H_o	375.0	feet
System static head	H_s	200.0	feet
Original head	H_1	340.0	feet
Original flow	Q_1	500.0	gpm
Starting speed	N_1	3560.0	rpm
Ending speed	N_2	3560.0	rpm
Starting impeller diameter	D_1	10.00	inches
Ending impeller diameter	D_2	10.75	inches
System Curve Exponent ($H=H_s+KQ^n$)	n	2.25	dimensionless
Pump Efficiency	Eff	0.75	dimensionless
Specific Gravity	SG	1.00	dimensionless
Original horsepower	HP_1	57.24	horsepower
Head predicted by affinity laws	$H_{affinity}$	392.91	feet
Flow predicted by affinity laws	$Q_{affinity}$	537.50	gpm
Horsepower predicted by affinity laws	$HP_{affinity}$	71.11	horsepower
Head expected under new system conditions	H_{system}	**388.26**	feet
Flow expected under new system conditions	Q_{system}	**576.61**	gpm
Horsepower expected under new system conditions	HP_{system}	**75.38**	Horsepower

Table 1

Affinity Laws for Pumping Systems

Once I realized this potential error, I decided to determine if the affinity laws could be modified so that they could be used to predict pumping system interactions. This quest became an obsession.

After numerous failed attempts, I was finally able to arrive at a simplified method to predict real-world pump and system interaction for the most general system curve relationship: $H = H_{static} + KQ^n$, where n can be any exponent. (Normally, n = 2, but any value can be used.)

The one key requirement was that I had to linearize the pump performance curve so that it has the form: $H = H_o + mQ$, where H_o is the point where a line tangent to pump curve at the location of interest (Q_1, H_1) intersects the vertical axis, i.e. the ordinate (see Figure 3). The slope "m" represents the instantaneous slope at the point of interest.

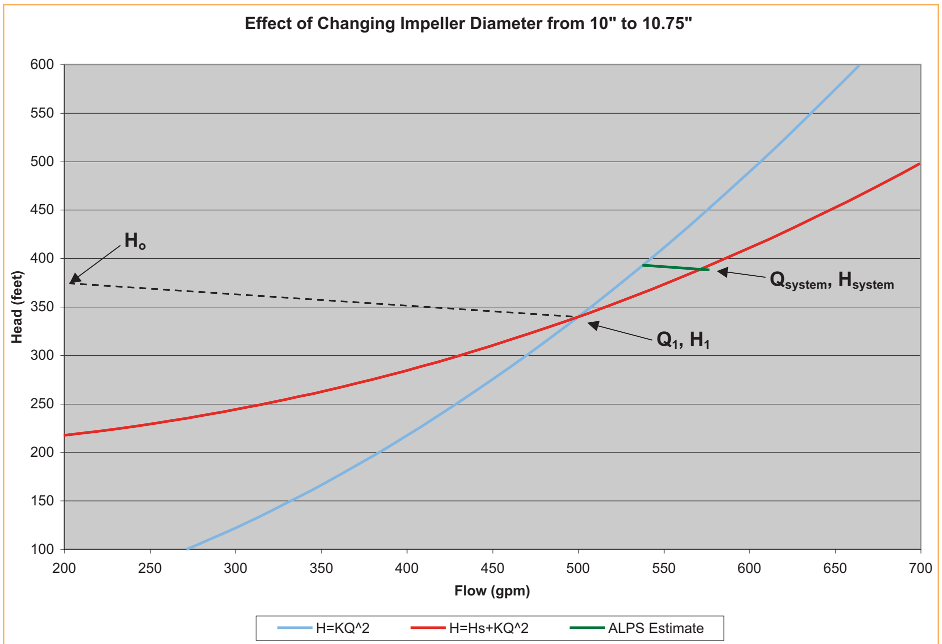


Figure 4

I also had to linerarize the friction only and general system curves at the point of interest. With these simplifications, I was able to arrive at solutions for Q_{system} and H_{system}, the expected operating point at the new conditions. More details of my analysis can be found in the appendix.

ALPS Example with Diameter Change

Let's first work through an example with a change in impeller diameter. Initially, we have to input all the required data in the lime-green boxes as shown in Table 1.

In this analysis, we have chosen to change the impeller diameter from 10-in to 10.75-in while maintaining constant pump speed. (Note: The input variable called "H_o" is that value where the pump curve tangent line intersects the Y-axis. In this case, its value is 375-ft.)

You can see that the head, flow, and horsepower values are different for affinity law predictions vs. the system curve predictions. If you look at Figure 4, you can see the ALPS prediction line (green) superimposed on for the affinity law curve and system curve.

Notice that the prediction made by ALPS is not perfect. It slightly passes the system curve. The ALPS prediction for flow is 576.61-gpm vs. 537.61-gpm for the affinity law prediction. Here, ALPS predicts a substantially greater flow that the traditional affinity laws!

Continuing this same example, let's see what effect this diameter change has on power consumption. You can see that power consumption increased from 57.24-hp to 75.38-hp, instead of the 71.11-hp the horsepower affinity law predicts. If you had a 75-hp motor driver, this disparity would make the difference between having a few horsepower of margin and running your motor at full load.

VFD Analysis Example

Now let's look at an example with changing speed. Assume we are changing speed from 3560-rpm to 3360-rpm, while maintaining a constant impeller diameter. Again, we first have to input our data into the ALPS calculation table (see Table 2).

With an assumed static head H_s equal to 200-ft, the values for head, flow, and horsepower are markedly different for the affinity law predicted values versus the ALPS predicted values. The system curve flow is estimated at 443.82-gpm and the flow for the affinity law prediction is 471.91-gpm. The graphical version of these results is shown in Figure 5.

Once again the ALPS prediction line does not exactly match the system curve. However, for small (<10 percent) changes in speed, the prediction is close enough for engineering purposes.

Description	Symbol	Value	Units
Vertical axis intersection of pump curve tangent line at point Q_1, H_1	H_o	375.0	feet
System static head	H_s	200.0	feet
Original head	H_1	350.0	feet
Original flow	Q_1	500.0	gpm
Starting speed	N_1	3560.0	rpm
Ending speed	N_2	3360.0	rpm
Starting impeller diameter	D_1	11.00	inches
Ending impeller diameter	D_2	11.00	inches
System Curve Exponent ($H=H_s+KQ^n$)	n	2.25	dimensionless
Pump Efficiency	Eff	0.75	dimensionless
Specific Gravity	SG	1.00	dimensionless
Original horsepower	HP_1	58.92	horsepower
Head predicted by affinity laws	$H_{affinity}$	311.78	feet
Flow predicted by affinity laws	$Q_{affinity}$	471.91	gpm
Horsepower predicted by affinity laws	$HP_{affinity}$	49.54	horsepower
Head expected under new system conditions	H_{system}	**312.08**	feet
Flow expected under new system conditions	Q_{system}	**443.82**	gpm
Horsepower expected under new system conditions	HP_{system}	**46.64**	Horsepower

Table 2

Another benefit of the ALPS methodology is its ability to determine if flow control by speed modulation is viable. Let's continue with the example, where speed is varied from 3560-rpm down to 3360-rpm. ALPS determines that system flow will drop from the original 500-gpm to 443.82-gpm, or a total of 56.18-gpm. This means we can expect 0.2809-gpm per rpm (56.18-gpm/200-rpm) change in pump speed.

If your VFD and motor system can control to increments of 20-rpm or less, this means (in theory) you can reliably control flow in increments as small as 5.62-gpm. If this is tolerable for your process, this means that VFD control can be used in place of a control valve. (Note: I have been told by VFD experts that speed can readily be controlled in increments of ±0.5 percent or less. In this example, 0.5 percent of 3600-rpm is about 18-rpm.)

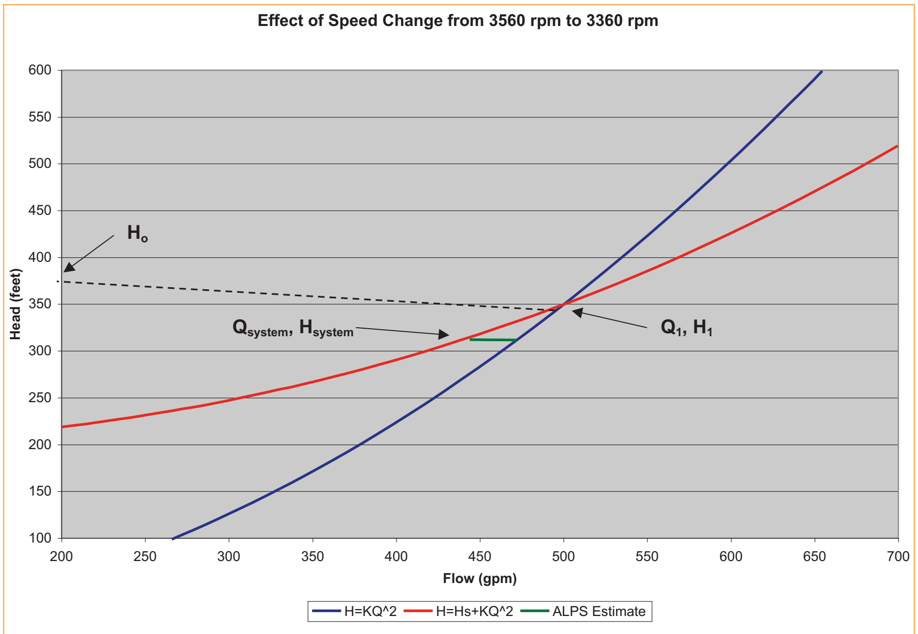


Figure 5

Caveats

You can see that ALPS is a powerful new tool for pump professionals, but it's not without its caveats. I want to clearly emphasize when and where ALPS should be used and when it shouldn't:

- ALPS can only be applied to a single pump "riding" on a system curve with the general form $H_s + KQ^n$. ALPS cannot handle the complication of a control valve in the pump's discharge line.
- ALPS is only valid for small (0 percent to 10 percent) changes in speed or impeller diameters, similar to the affinity laws.
- ALPS results are only estimates. If a more detailed analysis is required, you should graphically analyze the pump and systems curves.
- The analysis assumes you in a stable operating condition, i.e. no presence of cavitation, recirculation, etc.

Robert X. Perez, the website editor for PumpCalcs.com, has over 25 years of rotating equipment experience in the petrochemical industry, holds a BSME from Texas A&M University in College Station, a MSME degree from the University of Texas at Austin, a Texas PE license, and is an adjunct professor at Texas A&M University-Corpus Christi, teaching the Engineering Technology Rotating Equipment course. He can be reached at rxperez@pumpcalcs.com.

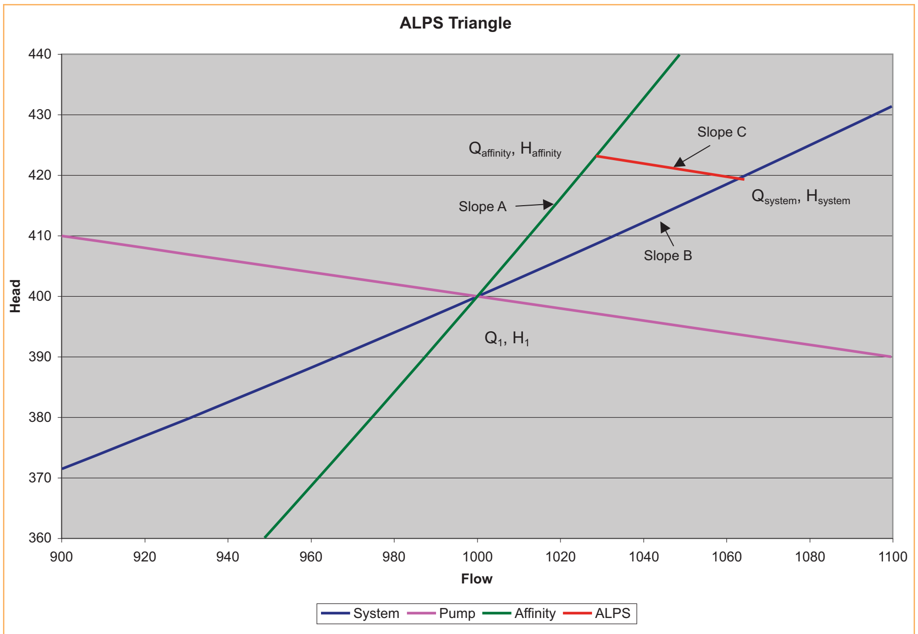


Figure A

Closing Comments

To use ALPS, simply visit PumpCalcs.com and proceed to the expert calculator titled "ALPS." This offers users a new means of analyzing VFD applications and investigating impeller change implications. I hope ALPS

(Continued on page 105)

(Continued from page 24)

finds its way into your pump analysis toolbox.

Appendix

Derivation of the ALPS Equations

The ALPS equation is based on the simple triangle that is formed by the linearized versions of the friction only system curve, friction + static head system curve and pump curve (see Figure A, page 24).

Point Q_1, H_1 of the triangle is the starting point. Point $Q_{affinity}$, $H_{affinity}$ is the where the linearized $H = KQ^2$ curve and linearized pump curve meet. Here, we assume the pump curve, in the simplified form $H = H_o + mQ$, is displaced upward according to the affinity laws while maintaining its same slope (m). Finally, point Q_{system}, H_{system} is where the system curve and the displaced pump performance line meet.

Next, we assign 1) the slope of the $H = KQ^2$ line at Q_1, H_1 the label A; 2) the slope of the system curve at Q_1, H_1 the label B; and 3) the slope of the linearized pump curve the label C.

We can solve for H_{system} and Q_{system} by starting with the following relationships (see Formulas).

Thus, we can state that by knowing a few basic pumping system parameters, we can <u>estimate</u> the effect of small pump speed or impeller diameter changes on system performance. The PumpCalcs.com expert calculator entitled "APLS" does all of this math for you.

P & S

References

1. Karassik, Igor, et al, *Pump Handbook*, New York, McGraw-Hill Book Company, 1976, pp 2.135-2.142.
2. Lobanoff, Val, Ross, Robert, *Centrifugal Pump – Design and Application*, Houston, Gulf Publishing, 1992, pp 12-14.

$$H_{affinity} = K_1Q^2 \Rightarrow \frac{dH_{affinity}}{dQ} = 2K_1Q$$

At Q_1, H_1, $A = \frac{dH_{affinity}}{dQ} = \frac{2H_1}{Q_1}$

$$H_{system} = H_s + K_2Q^n \Rightarrow \frac{dH_{system}}{dQ} = nK_2Q^{n-1}$$

At Q_1, H_1, $B = \frac{dH_{system}}{dQ} = \frac{n(H_1 - H_s)}{Q_1}$

$H_{pump} = H_o + CQ = H_o + CQ_1 = H_1$, therefore

$$C = \frac{H_1 - H_o}{Q_1}$$

We can now write:

$$A = \frac{H_{affinity} - H_1}{Q_{affinity} - Q_1} = \frac{dH}{dQ} = \frac{2H_1}{Q_1}$$

$B = \frac{H_{system} - H_1}{Q_{system} - Q_1} = \frac{dH}{dQ} = \frac{n \times (H_1 - H_s)}{Q_1}$, for the general form $H = H_s + KQ^n$

$$C = \frac{H_{system} - H_{affinity}}{Q_{system} = Q_{affinity}} = \frac{dH}{dQ} = \frac{H_1 - H_o}{Q_1}$$

$Q_{affinity} = Q_1 \times r$, where r is the impeller diameter or speed ratio

With four equations and four unknowns, we can readily solve for Q_{system} and H_{system}. Working through the math we arrive at the following relationship:

$Q_{system} = Q_1 \times \left[\frac{(B - A) - r(A - C)}{(B - C)}\right]$, where r is the impeller diameter or speed ratio

Simplifying further, we get:

$$Q_{system} = Q_1 \times \left[\frac{(n + r - 2)H_1 + rH_o - nH_s}{(n - 1)H_1 + H_o - nH_s}\right]$$

For the case where n=2, we can simplify even further:

$$Q_{system} = Q_1 \times \left[\frac{r(H_1 + H_o) - 2H_s}{H_1 + H_o - 2H_s}\right]$$

$$H_{system} = B(Q_{system} - Q_1) + H_1 = \frac{n(H_1 - H_s)}{Q_1} \times (Q_{system} - Q_1) + H_1$$

$HP_{system} = \frac{Q_{system}H_{system}SG}{3960\eta_1}$, where $\eta_1 \approx \eta_{system}$ for small changes in D or N

Formulas